The trend of k with increasing N is shown in Fig. 4 for each of these four loadings.

The sine-spline induced drag method has been in production use at Boeing for some time, and has given satisfactory results in all cases of which the author is aware. The method has proved useful in separating induced drag from lift-dependent drag measured by wind tunnel tests. It has also been used to calculate the induced drag of span loadings determined by theory. The method is coded in FORTRAN for the CDC 6600 digital computer, requires about 30,000 (octal) core locations, and requires about one second of CP time per case.

Acknowledgment

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Reference

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Airspeed Stability under Wind Shear Conditions

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Nomenclature

C_D = drag coefficient
h = height perturbation
m = mass
ρ = density
s = Laplace transform variable
S = wing area
t = time
T = thrust
τ = airspeed time constant
u, w = airspeed and vertical velocity perturbations
U_0 = equilibrium airspeed
X, Z = aerodynamic forces along X and Z axes
X_u^* = stability derivative $(\partial X/\partial u)/m$, with thrust
X_w = stability derivative $(\partial X/\partial w)/m$
Z_u^* = stability derivative $(\partial Z/\partial u)/m$, with thrust
Z_w = stability derivative $(\partial Z/\partial w)/m$

Introduction

A LANDING aircraft may encounter wind shear or a variation with altitude (and time) of the horizontal wind components along and normal to the landing approach path. The effect of the component along the path on altitude control has received recent attention because of aircraft accidents in which wind shear is a suspected cause. Investigations of wind shear effects usually have relied on large-scale computer simulations[1] or on the use of flight simulators. It is the purpose of this Note to point out that some insight into an aspect of the problem is furnished by a simple analytic model.

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Airspeed Stability

Horizontal wind strength component variations along an aircraft's landing path can create sudden altitude loss when the change is equivalent to a reducing head wind. Aircraft inertia will cause a sudden loss in head wind to result in the same loss in airspeed. Both lift and drag will decrease. The lift reduction will cause the aircraft to settle below the glide path, whereas the drag reduction will cause the aircraft to accelerate toward the original airspeed. The more rapid the airspeed recovery the less loss in altitude will result. Although the pilot can increase the acceleration toward the original airspeed by increasing thrust, it is pertinent to examine the aircraft's inherent tendency to regain airspeed, or its airspeed stability under wind shear conditions.

Mathematical Model

Airspeed stability in the sense described is found by a solution of the homogeneous linear differential equations of motion.[2] Pitch attitude perturbations are neglected on the assumption that the pilot or automatic pilot holds nearly constant attitude during the disturbance. Longitudinal and vertical velocity responses to an initial airspeed perturbation $u(0+)$ are, in Laplace transforms

$$u(s)/u(0+) = (s - Z_w)/\Delta \tag{1}$$

$$w(s)/u(0+) = Z_u^*/\Delta \tag{2}$$

where

$$\Delta = (s - X_u^*)(s - Z_w) - X_w Z_u^* \tag{3}$$

Conditions at the start of the motion described by Eqs. (1) and (2) correspond to the aircraft immediately after its airspeed has been perturbed by the amount $u(0+)$ from equilibrium, as by a sudden change in head wind.

The characteristic equation (3) factors into small and large real roots, for typical values of the stability derivatives. The small root may be called the "airspeed" root because it dominates the longitudinal motion governed by Eq. (1). The large real root contributes to the vertical or plunge motion. This root is equal approximately to $s - Z_w$. Time solutions of Eqs. (1) and (2) for a large jet transport in landing approach are shown in Fig. 1. The integral of the perturbed vertical velocity w also is shown. This is equivalent to a height perturbation h from the glide path. The airspeed perturbation u is almost a pure exponential. Under the assumptions, the height loss h approaches a fixed value of 4.53 ft/ft/sec of initial airspeed perturbation.

A single-degree-of-freedom representation of the disturbed longitudinal motion is suggested by the minor contribution to that motion of the large real root related to the vertical or plunge motion. The single-degree-of-freedom solution is

$$u(s)/u(0+) = 1/(s - X_u^*) \tag{4}$$

or

$$u(t)/u(0+) = e^{-t/\tau}$$

Table 1 Airspeed time constants for two commerical aircraft

	Jet transport	Light twin
Airspeed, kt	150.0	88.4
Gross weight, lb	245,400	3330
Wing area, ft^2	3123	178
C_D	0.167	0.090
$\partial T/\partial u$, lb/fps	−20.24	−0.74
Time constant, τ, sec	22.8	16.1

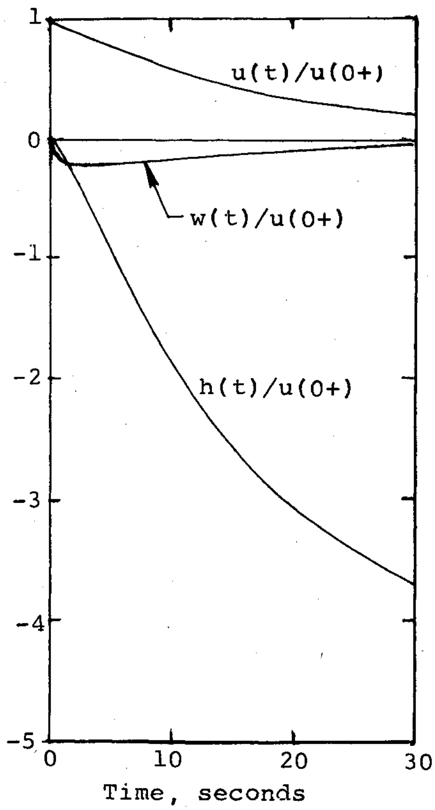

Fig. 1 Velocity and height perturbations, jet transport in landing approach.

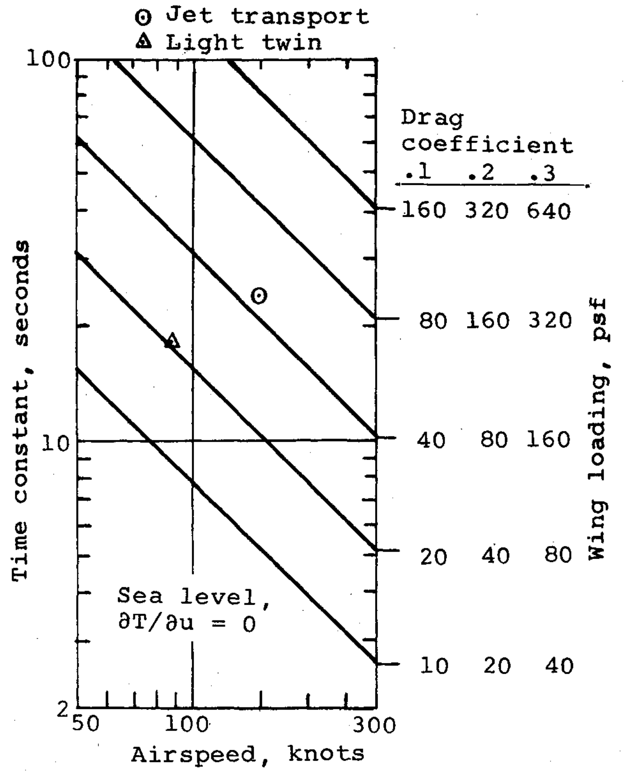

Fig. 2 Variation of airspeed time constant with drag, wing loading, and airspeed.

The root $s - X_u^*$ is an approximation to the "airspeed" root of Eq. (3). The time constant τ is thus an approximate "airspeed time constant." It is given by

$$\tau = -1/X_u^* \tag{5}$$

where

$$X_u^* = -(\rho S U_0 C_D/m) + (\partial T/\partial u)/m$$

Numerical Values

Airspeed time constants for two quite different commercial aircraft are derived for the full-flap landing approach configuration, at sea level, as appears in Table 1.

The light twin's airspeed time constant is about two-thirds that for the large jet transport. This should be reflected in less sensitivity to wind shear for the smaller aircraft. However, the airspeed time constants for both aircraft are long compared with pilot response time.

The thrust term in X_u^* and τ is only about 10% of the total. Thus, thrust can be neglected to obtain a useful generalized chart that shows the major influences on airspeed time constant, as in Fig. 2. Increased airspeed, lower wing loadings, and higher drag all produce favorable small time constants.

Conclusions

1) Small values of airspeed time constant provide an inherent rapid recovery of airspeed following drop in head wind under wind shear conditions.

2) Small values of airspeed time constant correspond to high airspeed, low wing loading, and high drag coefficient.

3) Landing approach with excess airspeed and in a high drag configuration, such as full flaps and speed brakes partially open, may be beneficial if wind shear is anticipated, to improve airspeed stability.

4) The airspeed time constant for a general aviation light twin aircraft is about two-thirds that of a commercial jet transport.

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Small Turbine Guide Vanes Loss Reduction

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Introduction

CURRENT applications show the need for advanced small gas turbine engines with high power/weight ratio, low specific fuel consumption, and small size. The helicopter and automotive gas turbine engines are examples of the need for such small engines.

Turbines used in small engines are characterized by their low weight, small size, and highly loaded blades. Losses in these turbines are large and cause a considerable reduction in turbine efficiency. Most of the existent experimental and analytical analyses of turbine losses are for large blade aspect ratio and are, therefore, not applicable to small, highly loaded turbines. The combination of the long chords together with

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